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TEST OF A MULTIPLYING MODEL FOR ESTIMATED AREA OF RECTANGLES

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Graphic ratings of physical size were obtained for 36 rectangles in a 6 × 6, width × height design, with each factor ranging from 3 to 18 cm. These judgments were approximately linear in physical area and followed a multiplying model reasonably well, though not perfectly. It is suggested that the underlying process was one of additive integration and that functional-measurement procedures can be used to scale phenomenal size of complex shapes.

Previous work on judgments of area has been largely concerned with accuracy, and with the relation between the response and the physical area (Ekman and Junge, 1961; Stevens and Guirao, 1963; Teghtsoonian, 1963; Stanek, 1969). This orientation toward the physical stimulus measure has tended to pass over the psychological processes involved in area judgments. The importance of a process orientation is suggested by reports that judged area appears to be affected by shape as well as by physical area (Luckiesh, 1922, p. 97; Warren and Pinneau, 1955; Smith, 1969). An interesting example is cited by Paterson and Tinker (1938; see also Helson and Bevan, 1964; Tolansky, 1964, p. 49), who noted that, contrary to appearances, the marginal white space on a printed page is roughly equal to the central printed area. In a typical page of text in this journal, for example, the marginal area occupies approximately 44% of the page.

The experiment reported here was designed to test a simple multiplying model for area of rectangles. The judged area was assumed to be, in effect, the product of the subjective values of width and height. A novel feature is the use of procedures from the theory of functional measurement (Anderson, 1970), procedures which test the model while allowing for subjective values of width and height. In this theory, no a priori assumption about the relation between subjective and objective stimulus values is needed. Differential weighting of the two stimulus dimensions is also allowed for, though this cannot be explicitly tested in a simple multiplying model.

METHOD

—Subjects and apparatus—The subjects were instructed to judge the actual physical area of rectangles using a linear graphic rating. The 36 basic rectangles were constructed from a symmetrical 6×6 factorial design. Both width and height of the rectangles varied from 3 cm to 18 cm in equal steps. The subject sat 1 m from the stimulus-response display and responded by varying the extent of exposed white part on an endless motorized strip of tape. The exposed white part could vary from 0 to 50 cm, and the left and right ends were anchored by squares of 1 and 23 cm respectively. The stimulus rectangle was presented midway between the two anchor cards and all these were in the same plane as the response tape. The subjects were 23 members of the student community, who were paid for their services.

—Materials and procedure—The rectangles were outlined in 1.5-mm black tape on square white tagboard cards, 23 cm on a side. In addition to the 36 stimuli, filler rectangles of 1 by 2, 1 by 3, 2 by 2, 18 by 20, 18 by 21, 20 by 21, 21 by 21, and 22 by 22 cm were included. Following 10 practice trials, these 44 stimulus rectangles were presented in a different shuffled order three times for each subject. The experimenter presented the stimuli, recorded the response, and reset the white tape to zero after each response.

RESULTS

The mean judgments are plotted as a two-way design in the left panel of Figure 1. Stimulus width is on the horizontal axis, and one curve is plotted for each of the six heights. The multiplying model implies that these curves should form a diverging fan of straight lines, and this is true to a good first approximation.

However, there is some discrepancy from the model, visible in the slight downward curvature of the bottom curve and in the more marked upward curvature of the top curve. A different view of this discrepancy is seen in the right-hand panel of Figure 1, which plots the response against the physical area on the horizontal axis. This shows small but definite deviations from a linear relation.

Almost the identical pattern of results was obtained in an earlier, unreported experiment with the same apparatus, design, and general procedure. The pattern thus appears to be reliable, but no explanation is known. A number of factors can probably be ruled out, however, and these require a few remarks.

The most important possibility is that the multiplying model applies but that subjective and objective values of width and height are not linearly related. Figure 1 is thus not a satisfactory test of the model, since it uses the physical values of the stimuli. However, a strong test

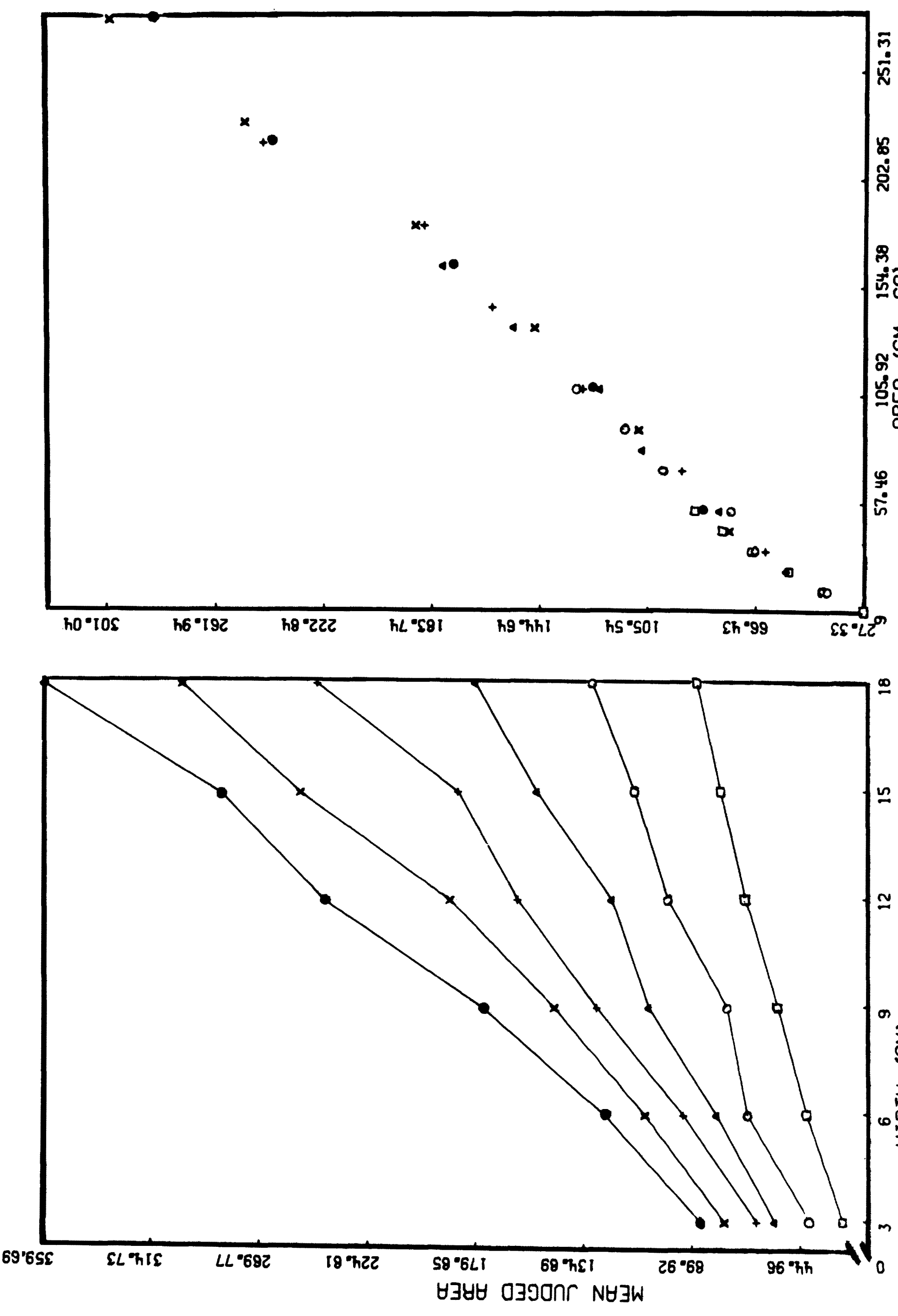

Fig. 1. Panel at the left shows mean judged area of 36 rectangles as a function of rectangle width on the horizontal; the six curves correspond to rectangles of heights 3, 6, 9, 12, 15, and 18 cm. Panel at the right shows mean judged area as a function of actual physical area on the horizontal; response was a linear graphic rating, with arbitrary zero and unit.

of the model is available, one which does not depend on the use of physical stimulus values (Anderson, 1970; Anderson and Shanteau, 1970) but instead rescales to the best estimates of their subjective values. The stimuli were rescaled in this way, but the resultant graph had the same overall shape, both for the present and for the earlier experiment. Accordingly, the discrepancy cannot be attributed to a nonlinear relation between subjective and objective values of width and height.

Other considerations

Four other factors deserve brief comment. First, assimilation-contrast effects between the rectangle and the card containing it can probably be ruled out. In the earlier experiment, each of 17 subjects judged three replications of the 36 rectangles on both 28-cm^2 cards and on 41-cm^2 cards. The graphs were almost identical for the two conditions.

Second, the judgments of the largest and smallest squares in Figure 1 seem perhaps too extreme. This might be an end effect in the response scale, which was suspected of being the source of the trouble, since similar results had appeared in the earlier experiment. The present filler rectangles, outside the range of the 36 experimental rectangles, were included to eliminate such an effect. Thus, the reappearance of the same pattern suggests that it is not the result of an end effect in the response scale.

Third, there might be shape effects, so that rectangles of the same area but different dimensions would produce different responses. Previous reports (Smith, 1969) have suggested that figures with one large and one small dimension tend to be judged larger (though no direct test seems to have been made). The present design included a direct comparison. Inspection of the left panel of Figure 1 shows a larger response to the 3 by 18 and 18 by 3 rectangles than to the 6 by 9 and 9 by 6 rectangles. A similar, slightly larger effect was also found in our earlier experiment, and both were significant by post hoc tests [$F = 6.72$ and 4.43, $df =$ 1/16 and 1/22, for the previous and present experiments]. One other direct comparison is given by the 6 by 18 and 9 by 12 rectangles, for which a similar shape effect was observed in both experiments—though it is very slight in Figure 1 (left panel). Such a shape effect could account for the upward curvature of the top curve there and would also be consistent with the fact that the judgments of the squares fit well to a quadratic curve. However, the slight downward curvature in the bottom curve would remain unexplained.

Finally, there is the ever-present possibility that the response scale is subject to a small nonlinear response bias. The present data unfortunately provide no information on this, but it may be desirable to use two somewhat different response scales in future work.

DISCUSSION

The discrepancy from the multiplying model should not obscure the reasonably good fit that it gives to the data. It is appropriate, therefore, to inquire more closely into the process underlying the judgments. Teghtsoonian (1965) reports that subjects judged the physical area of squares and circles by estimating and squaring a linear dimension. For rectangles, analogously, they could estimate width and height and multiply the two, though the subjects in our experiment did not report doing so.

An alternative, perhaps simpler, hypothesis is that the process is one of additive integration. Responses obeying an additive process would still appear to follow a multiplying model. This additive hypothesis also applies to irregular figures more directly than would a multiplying model. Accordingly, it may merit consideration even though there is some evidence against it. In its simplest form, it would imply that judgments of physical area should be independent of shape. Small shape effects, such as obtained here, might not be a serious problem, since shape variables might be expected to influence the integration to some degree. However, two other reports seem to indicate somewhat more substantial deviations from the additive hypothesis. The first is the page-margin effect mentioned in the introduction. The second is Teghtsoonian's report (1965) that judgments of both apparent size and of physical area for irregular polygons followed a power function with exponent of about .8.

A clear distinction must, of course, be made between judgments of size and area. Size can refer to the linear dimensions of a figure as well as to its area, and subjects may not always make a clear distinction between the two, even when told to judge physical area. The page-margin effect, for example, seems in part to result from a comparison of linear dimensions. Both linear and areal cues could then contribute to the response, perhaps as a weighted average.

Functional-measurement procedures (Anderson, 1970) might provide a useful approach. Stimulus figures could consist of two or more spatially separated components, combined according to factorial designs. If the adding hypothesis has any validity, then judgments of total area should be additive in the stimulus components. This could hold even though the components themselves were judged and evaluated on a different basis —and showed shape effects, for instance. The hypothesis of component addition can be tested simply and directly using the raw response measure. If the hypothesis holds, then the subjective values of the components can be estimated directly from the data on equal interval scales.[1] This would be an important result, since it would give a theoretically valid measure of the effective phenomenal size of the components. Such measures would provide a solid base for the study of the processes underlying the valuation of the components.

Notes

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1. Functional scaling of length is illustrated in Weiss and Anderson, 1969.

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